UNNS Spectral Geometry of the Mandelbrot and Beyond

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Abstract

The Mandelbrot set, classically a subset of the complex plane, reveals deeper structure when interpreted through the lens of the UNNS substrate. By embedding iteration depth as a recursion axis, the Mandelbrot and Julia sets are recast as recursive geometries with associated spectral invariants. We define UNNS entropy, spectral gaps, and echo boundaries, linking these to concepts in quasicrystal tilings, Lyapunov stability, and wave propagation. The result is a spectral geometry of recursion that bridges fractals, dynamical systems, and discrete field theories.

Contents

1 Introduction

2 Applications and Implications

- **Complexity:** Escape entropy as a measure of algorithmic complexity.
- **Physics:** Recursive lattices as analogs for quasicrystals and photonic structures.
- **Mathematics:** UNNS spectral invariants connect dynamics, number theory, and tilings.

3 Conclusion

The Mandelbrot set, viewed through UNNS, becomes more than a planar fractal: it is a recursion-geometry object with spectral invariants, entropy surfaces, and quasicrystal analogs. This perspective links fractals with tilings, waveguides, and recursion theory, forming the basis of a UNNS spectral geometry.

A Worked Numerical Examples

A.1 Fibonacci Recurrence and Spectral Lattice

The Fibonacci recurrence

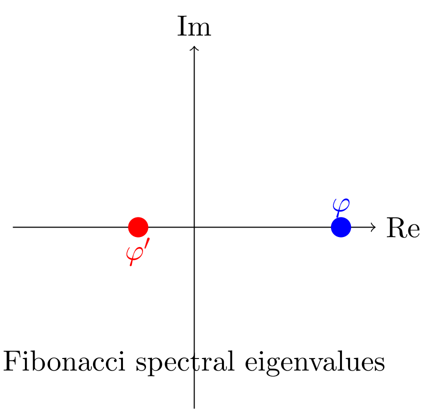
$$F_{n+1} = F_n + F_{n-1}$$

can be represented by the matrix

$$A = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}.$$

Proposition 1. *The eigenvalues of A are $\varphi = \frac{1+\sqrt{5}}{2}$ and $\varphi' = \frac{1-\sqrt{5}}{2}$, corresponding to the golden ratio and its conjugate.*

These eigenvalues define the spectral lattice of Fibonacci recursion, which is the basis for Penrose tilings and quasicrystal diffraction.



Fibonacci spectral eigenvalues

A.2 Escape Entropy and Lyapunov Comparison

For c outside the Mandelbrot set, define escape time $\tau(c)$:

$$S(c) = \log \tau(c).$$

For the same orbit, the Lyapunov exponent is

$$\lambda(c) = \lim_{n\to\infty} \frac{1}{n} \sum_{k=0}^{n-1} \log |f'(z_k)|.$$

Remark 1. *$S(c)$ measures recursion duration before divergence, while $\lambda(c)$ measures exponential divergence rate. Their correlation is a UNNS spectral invariant.*

A.3 Spectral Gaps from Recursive Orbits

Let $\{z_n\}$ be a recursive orbit. Its discrete Fourier transform is

$$\hat{z}(\omega) = \sum_{n=0}^{N} z_n e^{-i\omega n}.$$

Zeros of $\hat{z}(\omega)$ correspond to spectral gaps. These mirror forbidden frequencies in quasicrystals and photonic band gaps.

A.4 Example Pseudocode

Listing 1: Compute escape entropy and Lyapunov exponents for quadratic iteration

```python
import numpy as np

def escape_entropy(c, max_iter=500, R=2):
    z = 0
    for n in range(max_iter):
        z = z*z + c
        if abs(z) > R:
            return np.log(n+1)  # UNNS entropy
    return np.inf

def lyapunov_exponent(c, max_iter=500):
    z = 0
    lam = 0
    for n in range(max_iter):
        z = z*z + c
        lam += np.log(abs(2*z)+1e-8)
    return lam / max_iter

# Example usage:
c = 0.3 + 0.6j
print("UNNS␣Escape␣Entropy:", escape_entropy(c))
print("Lyapunov␣Exponent:", lyapunov_exponent(c))
```

This pseudocode allows readers to compute UNNS entropy and Lyapunov exponents for sample parameters c.

B Discussion of Numerical Results

- Fibonacci eigenvalues connect UNNS recursion to quasicrystal order.
- Escape entropy $S(c)$ and Lyapunov $\lambda(c)$ serve as dual UNNS invariants.
- Spectral gaps explain resonance and suppression phenomena in recursive lattices.

References

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[3] D. Shechtman et al., *Metallic Phase with Long-Range Orientational Order and No Translational Symmetry*, Phys. Rev. Lett. 53, 1984.

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